

Signifying and meaning-making in mathematical thinking, teaching, and learning

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Not meaning, not consciousness lies behind life, but life lies behind consciousness.
Leont'ev (2009, p. 96)

1 Introduction

In the last decades, concepts of learning have changed dramatically. The classical concept of learning as the simple reproduction of given conceptual contents has yielded space to a modern concept that emphasizes the creative and critical involvement of the students. Some recent theoretical accounts add a symmetrical stance and claim that, in highly social and cultural organized institutional settings, such as the school, learning cannot be abstracted from teaching. Teaching entails not only a critical stance toward the teachers' own teaching but also a fundamental and creative involvement in the students' act of learning. In those accounts, teaching and learning appear as two sides of the same coin: They are considered as part of a same process, connected by interrelated processes of signifying and meaning-making—that is to say, processes of communication and mutual understanding. Underneath them rests a unifying semiotic concept: meaning.

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As evidenced by the number of books and articles devoted to elucidating its nature, meaning has proved to be a difficult concept. Now, regardless of how we conceptualize it, it is clear that, to be expressed, meaning can come into presence through signs only—be they pointing gestures, utterances, texts, or succinct mathematical formulas. Meaning, that which we intend to express or communicate (for others and perhaps for ourselves too), has to be *sign-ified*. “Meaning,” Vološinov (1973, p. 28) argued, “is not a thing and cannot be isolated from the sign as if it were a piece of reality existing on its own apart from the sign.” This point bears particular importance for mathematics education, since mathematics is an intrinsically symbolic activity—something that we accomplish through written, oral, bodily, and other signs. And, of course, because of its focus on signs and meanings, semiotics is well-suited to investigate mathematics teaching and learning processes.

It would be inaccurate to think, however, that semiotics studies signs *as such*, formally. Semiotics brings with it *theoretical* accounts of how signs signify. Semiotics offers accounts of how meanings work, how subjects produce and use them, and also how, in producing them, subjects become subjects of meaning. Semiotic perspectives provide us with theoretical approaches and the ensuing methodological tools to investigate human action and, in our case, the teaching and learning of mathematics. The variety of possible semiotic accounts can be exemplified by referring to Peircean, Vygotskian, Piagetian, Saussurean, and post-modern semiotics (Radford, 2006). These semiotic approaches offer mathematics educators forms of action and understanding.

2 Meaning as *praxis*

Although different in their principles, current semiotic perspectives might be characterized by a move from substantialist principles to relational ones (see, e.g., Brier, 2008; Marková, 2003; Stawarska, 2009). What this shift means is that the MEANINGS of concepts (mathematical, linguistic, and otherwise) are not necessarily conceived of as referring to something “objective” in the world, but as something embedded in the social and cultural practices in which they evolve. This is the point that Bronislaw Malinowski, one of the most influential twentieth century anthropologists, was led to make many years ago in his observation of language in cultures different from his own. As he put it,

Since the whole world of ‘things-to-be-expressed’ changes with the level of culture, with geographical, social and economic conditions, the consequence is that the meaning of a word must be always gathered, not from a passive contemplation of this word, but from an analysis of its functions, with reference to the given culture. (Malinowski, 1923, p. 309)

Malinowski’s idea of meaning as *praxis* was articulated later on by Wittgenstein (1953), who suggested that the meaning of words can only be grasped in terms of their use in language. Meanings live in *language games*, in forms of life. These language games set the limits of what can be experienced and made expressible: They enable and restrict us in how we experience our surrounding world and speak about desires and intentions. This is why

experience exists even for the person undergoing it only in the material of signs. Outside that material there is no experience as such. In this sense *any experience is expressible*, i.e., is potential expression. Any thought, any emotion, any willed activity is expressible. This factor of expressivity cannot be argued away from experience without forfeiting the very nature of experience. (Vološinov, 1973, p. 28; italics in the original)

Because of the inseparability of signs and subjective experience, the very texture of human consciousness is made up of signs. And, as Vološinov contended, consciousness can become consciousness only in the process of social interaction. In other words, like meaning, consciousness is dialogical, that is to say, other-oriented.

The semiotic nature of meaning and consciousness leads us to enquire about the manner in which signs circulate in a culture and are made accessible (or not) to its individuals. Two aspects are salient in this respect: the historical and the political dimension of meanings.

3 Meaning as *historical* and *political* praxis

Meaning as praxis may be conceptualized in different ways. For instance, we can assume that the concepts we signify through signs are mere names without any correspondence to reality. This way of theorizing has its ancestor in the well-known Medieval nominalism. We can also assume that signs signify in terms of structural differences. This line of enquiry draws on Ferdinand de Saussure's (1916) linguistics and was developed in structural terms later, in the first half of the twentieth century in various research fields. Claude Lévi-Strauss (1958, 1962) is a prominent example in the field of anthropology, while in the investigation of conceptual growth there is certainly no better example than Piaget's (1970, 1971) work. The structural movement assumed that the structures under investigation (social organizations, in the case of anthropology; intellectual development, in Piaget's case) are manifested in terms of organizing rules and mechanisms beyond the relative particularity of cultures. In other accounts, however, meaning is thought of as something that cannot be grasped in formal structural terms. Meaning needs to be related to the concrete historical, cultural context. It is here that we find Foucault's archeology of knowledge and its efforts in *historicizing* meaning (Foucault, 1969). It is also here that we find Vygotsky's historical-cultural school of thought and its emphasis on the fact that an account of meaning as praxis should include a clear reference to the concrete reality that people build through their interactions. Meaning acquires here a specific functional status: It is overall "an inseparable part of the word" (Vygotsky, 1987, p. 47) and the sign more generally. It appears as a joint generalized reflection of reality—or, more precisely, as the crystallized generalization of the activity of reflecting on an unfolding reality that we produce and the place we, as human subjects, come to occupy therein. And because the continuous unfolding of what we take to be our reality is not something that starts with us but has started before our arrival into the world, meaning is not a mere praxis: It is a *historical* praxis.

As Leont'ev suggested,

Meaning is that generalization of activity that is crystallized, fixed in that sensitive carrier of it [i.e., signs] . . . This is an ideal . . . crystallization of social experience, the social practice of humanity. The circle of ideas of a given society, its science, its very language, all of this is the essence of the system of meaning. (Leont'ev 1978, p. 169)

In this account, meaning belongs to that circle of ideal phenomena, more precisely, "the phenomena of social consciousness" (Leont'ev, 1978, p. 171), something that Evald Ilyenkov explains in the following terms:

social consciousness is not simply the many times repeated individual consciousness (just as the social organism in general is not the many times repeated individual human organism). [Social consciousness] is, in fact, a historically formed and historically developing system of 'objective notions', forms and patterns . . . This

system comprises all the general moral norms regulating people's daily life, legal precepts, the forms of state-political organization of life, the ritually legitimized patterns of activity in all spheres. (Ilyenkov, 1977, p. 77)

As a historical crystallized generalization of activity, meaning, much as culture, is also the space of conflicting views and oppositions. Meaning is tension. Meaning brings with it worldviews and the historical sediments of class struggle and dissidence. What is important in words and signs in general, Vološinov (1973, p. 19) tells us, is not so much their purity to signify, but their social ubiquity. "The word is implicated in literally each and every act or contact between people—in collaboration on the job, in ideological exchanges, in the chance contacts of ordinary life, in political relationships, and so on." In short, meaning is historical and political. This is why mathematics education can no longer be seen as the innocent transmission and diffusion of mathematical knowledge (Radford, 2011). Miguel, Vilela, and Lanner de Moura (2010) note how, in subtle ways, school mathematics conveys (and promotes) the encyclopedic ideas of abstraction, generalization, rigor, quantification, reduction, and simplification. The problem, of course, is not merely the choice and commitment that school mathematics makes in the types of meanings it endeavors to promote. The challenge is to become sensitive to the choices, so as to open a space in order to try to see underneath them and, hopefully, to critically transform meaning and its cultural forms of production.

4 Sense and meaning

To assert that meaning is an ideal form of reflection does not amount to conceiving of it as something Platonic: "Meanings do not have existence except as in the consciousness of concrete people. There is no independent kingdom of meanings, there is no Platonic world of ideas" (Leont'ev, 1978, p. 169).

Leont'ev also repeatedly underlines that meaning has a double nature, a duality that is extremely important to understand. Referring to the duality of the existence of meanings he says:

This duality lies in the fact that meanings present themselves to the subject both in their independent existence—as objects of his consciousness—and at the same time as the means and 'mechanism' of comprehension, that is, when functioning in processes that present objective reality to the subject. In this function meanings necessarily enter into internal relationships linking them with other 'formative elements' of the individual consciousness; it is only in these internal systemic relationships that they acquire *psychological* characteristics[.]

Let us put this in a different way. When the products of socio-historical practice, idealised in meanings, become part of the mental reflection of the world by the individual subject, they acquire new systemic qualities. The major difficulty here is that meanings lead a double life. They are produced by society and have their history in the development of language, in the history of the development of forms of social consciousness; they express the movement of science and its means of cognition, and also the ideological notions of society—religious, philosophical and political. In this objective existence of theirs, meanings obey the socio-historical laws and at the same time the inner logic of their development.

However, despite all the inexhaustible wealth, all the diversity of this life of meanings (this is what all the sciences are about), there remains hidden within it another life and

another kind of motion—their functioning in the processes of the activity and consciousness of specific individuals, even though they can exist only by means of these processes. In this second life of their meanings are individualized and ‘subjectivized’ only in the sense that their movement in the system of social relations is not *directly contained* in them; they enter into another system of relationships, another movement. But the remarkable thing is that, in doing so, they do not lose their socio-historical nature, their objectivity. (Leont’ev, 2009, p. 18; italics in the original)

This dual nature of meaning is a product of the history of culture, resulting in the problem that objective and subjective forms of meaning will never coincide:

As we have already said, consciousness owes its origin to the identification in the course of labour of actions whose cognitive results are abstracted from the living whole of human activity and idealised in the form of linguistic meanings. As they are communicated they become part of the consciousness of individuals. This does not deprive them of their abstract qualities because they continue to imply the means, objective conditions and results of actions regardless of the subjective motivation of the people’s activity in which they are formed. At the early stages, when people participating in collective labour still have common motives, meanings as phenomena of social consciousness and as phenomena of individual consciousness directly correspond to one another. But this relationship does not endure in further development. It disintegrates along with the disintegration of the original relationships between individuals and the material conditions and means of production, along with the emergence of the social division of labour and private property. The result is that socially evolved meanings begin to live a kind of double life in the consciousness of individuals. Yet another relationship, another movement of meanings in the system of the individual consciousness is brought into being. (Leont’ev, 2009, p. 20)

The relationship between meaning as a historical–cultural construct and its mode of existence in the consciousness of concrete people led Leont’ev to distinguish between *meaning* and *sense*. While the former, as previously noted, is a historical and ideal entity, the latter is personal, subjective. Their relationship cannot be cast in terms of logical versus psychological but must be considered in *dialectical* terms: as a relationship between the general and the individual. Naturally, in developmental terms, sense, like meaning, evolves. Sense evolves as individuals engage in goal-oriented activities. But sense cannot be taught. It can only be nurtured. For sense is not a thing but a relationship: the relationship between the actions that realize the individual’s activity and the motives of this activity.

Thus, that which I actually recognize, how I recognize it, what kind of sense what I recognize has for me is determined by the motive of activity in which my specific action is included. For this reason the question of sense is always a question of motive. (Leont’ev, 1978, p. 173)

Sense refracts historical and political meanings through the prism of the individual’s own biography, projects of life, and aspirations.

Through its various theoretical approaches, semiotics offers interesting perspectives through which to investigate and consider the relationship of meaning and sense, theoretically and empirically. One of the important contributions of contemporary semiotics to education is, indeed, to cast the problem of meaning and sense in ways that go beyond the realm of speech and language. Without dismissing the central role of

language in human cognition, semiotic approaches acknowledge the crucial role of other important aspects in signifying and meaning-making processes. Embodiment is one of them. This “corporeal turn” (Sheets-Johnstone, 2009) is neither a mere regress to the French eighteenth century interest in sensations nor a return to the seventeenth century British empiricism. It is rather the acknowledgment of the amazing spectrum of resources to which individuals resort in trying to make sense of the events of quotidian life. The interest in embodiment makes new perspectives available to investigate the nature of mathematical thinking.

5 Thinking and emotions

At the end of chapter 1 of *Thinking and Speech*, Vygotsky asks the question: “Is word meaning speech or is it thought?” His answer is as follows: “It is both at one and the same time; it is a *unit of verbal thinking*” (Vygotsky, 1987, p. 47; emphasis as in the original).

However, thinking in general and mathematical thinking in particular resort not only to verbal thinking but also to diagrammatic or iconic (Peirce, 1931–1958) and other forms of embodied thinking (Merleau-Ponty, 1945). We could hence reformulate Vygotsky’s question as asking whether sign meanings are just outer forms of sign activity or part of thought itself. And we can answer the question in the same way as Vygotsky did: It is both. The inclusion of embodied forms of semiotic action in the realm of thinking allows for signifying and meaning-making processes to be considered as deeply related to sensuous forms of doing and reflecting (Radford, 2009). Thinking turns out to be a dynamic *unity of material and ideal components*—a tangible social practice materialized in the body, in the use of signs, and artifacts of different sorts.

By including an embodied approach, semiotic perspectives move hence away from traditional accounts in which thinking is considered to be something purely mental. But they also open up new spaces to move away from traditional accounts in which thinking is considered to be a kind of “cold” processor of ideas and representations. Semiotic perspectives provide room to conceive of emotions as constitutive parts of thinking. This is the point that Vygotsky was making in his investigations about the genetic roots of thinking and speech. In trying to elucidate the meaning of the child’s first words, Vygotsky refers to Stern: “The childish *mama*, translated into advanced speech, does not mean the word ‘mother’ but rather a sentence such as ‘Mama, come here,’ or ‘Mama, give me,’ or ‘Mama, put me in the chair,’ or ‘Mama, help me’” (Stern, quoted in Vygotsky, 1962, p. 30; italics in the original).

While agreeing with Stern about the emotional dimension of the emerging language and its direction toward concrete objects in the environment of the child, Vygotsky found, however, that Stern’s account failed in taking into account the whole behavior of the child, in particular the child’s behavior in its concrete situation:

When we observe the child in action, however, it becomes obvious that it is not only the word *mama* which means, say, ‘Mama, put me in the chair,’ but *the child’s whole behavior at that moment* (his reaching out toward the chair, trying to hold on to it, etc.). (Vygotsky, 1962, p. 30; italics in the original)

Thus, the emotional dimension is not just encapsulated in the word; it is conveyed by the whole range of actions (grasping, holding, gesturing) as presented in the whole situation. Stern’s account, Vygotsky argued, portrayed a form of intellectualizing conception of emotions. Vygotsky’s point was that emotions are embodied drivers of thinking expressed in various ways.

Recent neurological research is consonant with Vygotsky's view on emotions. As neurologist Antonio Damasio puts it, "Emotions are not a dispensable luxury . . . Old as emotions are in evolution, they are fairly high-level component of the mechanisms of life regulation" (1999, p. 54). One of the patients studied by Damasio—patient Elliot, a "pleasant and intriguing thoroughly charming but emotionally contained person" (2005, p. 34)—displayed intact linguistic and intellectual skills. Yet, Elliot, who underwent the extraction of a tumor that resulted in damages to prefrontal cortices, showed an impaired ability to reach decisions. This new cognitive impairment was accompanied by perturbations in the emotional sphere, which resulted in his disaffected experiences of positive or negative emotions. As he confessed to the neurologist, "his own feelings had changed from before his illness" (2005, p. 45). Without the high-level emotional component of life regulation, thinking was disrupted. Damasio suggests that we "imagine this emotional component [of life regulation] as sandwiched between the basic survival kit (e.g., regulation of metabolism; simple reflexes; motivations; biology of pain and pleasure) and the devices of high reason" (1999, p. 54). Hence, far from being a dispensable luxury, emotions are key components of thinking and the activities we engage in: "Emotions are not subordinated to activity but appear to be . . . the 'mechanism' of its movement" (Leon'tev, 1978, p. 120).

The abstractness of meaning and emotional stiffness in mathematics learning is often a challenge that is difficult to meet for children in school. Meaning-making is the difficult development of subjective meaning—subjective not in the solipsistic or individualist sense of traditional interpretations of subjectivity, but meant as the individual co-appropriation of the objective cultural world.

Two important issues that spring from this understanding of meaning deserve our strongest interest and attention. First, in mathematics learning, it is not easy to find how meaning is related to the world of objects partly because the meaning is not related to more or less material objects but to the activity itself. The key to meaning-making can only be to base it on the development of a *communal* practice of meaning-making. The abstractness and strangeness of mathematical meaning can only rarely be overcome through appealing to reasoning. Second, meaning-making has to be understood as a developmental process. As the infant beginning to speak, in an example of extreme abstract reasoning, uses the concept "dog" for all creatures on four legs, so the beginner in mathematics learning finds it hard to understand how abstract and how concrete mathematical meaning should be. Seeing the development of meaning-making in focus consists not only of understanding meaning-making as developing (as opposed to as given). It means also, in the tradition of Vygotsky's idea of the unity of development and teaching, that teaching should march ahead of the development of meaning and create zones of proximal development where students make meaning.

The articles of this Special Issue address some of the topics mentioned above. Some of them explore the semiotic activity in classroom processes of signifying and meaning-making. Other papers deal with epistemological and conceptual issues about learning, mathematical structures, and the evolution of symbolic notations from a semiotic perspective. Most of the articles are expanded versions of papers presented at the Third Conference of the Semiotic Approach to Mathematics, the History of Mathematics and Mathematics Education, which took place on July 16 and 17, 2009 in Thessaloniki, Greece. The Special Issue is the continuation of previous research by the "Semiotics Group" whose previous work appeared in the book *Semiotics in mathematics education: epistemology, history, classroom, and culture* (Radford, Schubring, & Seeger, 2008). We thank Michael Fried for agreeing to write a postscript to the Special Issue introduced here and Marianna

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References

- Brier, S. (2008). *Cybersemiotics*. Toronto, ON, Canada: Toronto University Press.
- Damasio, A. (1999). *The feeling of what happens*. San Diego, CA: Harcourt.
- Damasio, A. (2005). *Descartes' error. Emotion, reason, and the human brain*. New York: Penguin Books.
- Foucault, M. (1969). *L'archéologie du savoir [Archeology of knowledge]*. Paris, France: Éditions Gallimard.
- Ilyenkov, E. (1977). The concept of the ideal. In R. Daglish (Tr.), *Philosophy in the USSR: Problems of dialectical materialism* (pp. 71–99). Moscow, Russia: Progress.
- Leont'ev, A. N. (1978). *Activity, consciousness, and personality*. Englewood Cliffs, NJ: Prentice-Hall.
- Leontyev [or Leont'ev], A. N. (2009). *Activity and consciousness*. Pacifica, CA: MIA. Retrieved August 29, 2009, from <http://www.marxists.org/archive/leontev/works/activity-consciousness.pdf>.
- Lévi-Strauss, C. (1958). *Anthropologiestructurale [Structural anthropology]*. Paris, France: Plon.
- Lévi-Strauss, C. (1962). *La pensée sauvage [The savage mind]*. Paris, France: Plon.
- Malinowski, B. (1923). The problem of meaning in primitive languages, supplement 1. In C. K. Ogden & I. A. Richards (Eds.), *The meaning of meaning* (pp. 296–336). New York: Harcourt, Brace, & Co.
- Marková, I. (2003). *Dialogicality and social representations*. Cambridge, MA: Cambridge University Press.
- Merleau-Ponty, M. (1945). *Phénoménologie de la perception [Phenomenology of perception]*. Paris, France: Gallimard.
- Miguel, A., Vilela, D., & Lanner de Moura, R. (2010). *Desconstruindo a matemática escolar sob uma perspectiva pós-metafísica de educação [Deconstructing school mathematics from a post-metaphysical perspective on education]*. *Zetetiké*, 18, 129–205.
- Peirce, C. S. (1931–1958). *Collected papers* (Vol. I–VIII). Cambridge, MA: Harvard University Press.
- Piaget, J. (1970). *Genetic epistemology*. New York: Norton.
- Piaget, J. (1971). *Structuralism*. New York: Harper & Row.
- Radford, L. (2006). *Semiótica y educación matemática [Semiotics and mathematics education]*. *Revista Latinoamericana de Investigación en Matemática Educativa*, Special Issue: *Semiótica, cultura y pensamiento matemático [Semiotics, culture, and mathematical thinking]*, 7–21.
- Radford, L. (2009). Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings. *Educational Studies in Mathematics*, 70(2), 111–126.
- Radford, L. (2011). La evolución de paradigmas y perspectivas en la investigación. El caso de la didáctica de las matemáticas [The evolution of paradigms and perspectives in research. The case of mathematics education]. In J. Vallès, D. Álvarez, & R. Rickenmann (Eds.), *L'activitat docent intervenció, innovació, investigació [Teacher's activity: Intervention, innovation, research]*. Girona, Spain: Documenta Universitaria.
- Radford, L., Schubring, G., & Seeger, F. (2008). *Semiotics in mathematics education: Epistemology, history, classroom, and culture*. Rotterdam, The Netherlands: Sense.
- Saussure, F. (1916). *Cours de linguistique générale [Lectures on general linguistics]*. Paris, France: Payot.
- Sheets-Johnstone, M. (2009). *The corporeal turn*. Exeter, UK: Imprint-academic.com.
- Stawarska, B. (2009). *Between you and I. Dialogical phenomenology*. Athens, OH: Ohio University Press.
- Vološinov, V. N. (1973). *Marxism and the philosophy of language*. Cambridge, MA: Harvard University Press.
- Vygotsky, L. S. (1962). *Thought and language*. Cambridge, MA: MIT.
- Vygotsky, L. S. (1987). Thinking and speech. In R. W. Rieber & A. S. Carton (Eds.), *Collected works* (Vol. 1). New York: Plenum.
- Wittgenstein, L. (1953). *Philosophical investigations*. Oxford, UK: Blackwell.